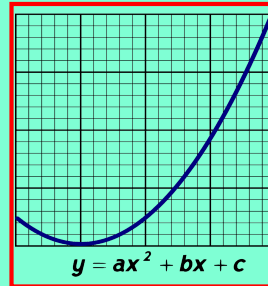


Math 125
Fall 2021
Lecture 40



Solve

$$\begin{cases} x^2 - 2y = 10 \\ 3x - y = 9 \end{cases}$$

→ Isolate y

$$-y = 9 - 3x \Rightarrow y = 3x - 9$$

Making Subs. $\Rightarrow x^2 - 2(3x - 9) = 10$

$$\begin{aligned} y &= 3x - 9 \\ &= 3(2) - 9 \\ &= 6 - 9 \\ &= -3 \end{aligned}$$

$$\begin{aligned} y &= 3x - 9 \\ &= 3(4) - 9 \\ &= 12 - 9 \\ &= 3 \end{aligned}$$

$$x^2 - 6x + 18 - 10 = 0$$

$$x^2 - 6x + 8 = 0$$

$$(x - 2)(x - 4) = 0$$

$$x = 2 \quad x = 4$$

$$y = -3 \quad y = 3$$

$$\{(2, -3), (4, 3)\}$$

Ex:

Solve

$$\begin{cases} y = x^2 + 1 \\ 4x - y = -1 \end{cases}$$

Using Subs. Method

$$4x - (x^2 + 1) = -1$$

$$4x - x^2 - 1 = -1$$

$$y = x^2 + 1$$

$$= 0^2 + 1 = 0 + 1 = 1$$

$$y = x^2 + 1$$

$$= 4^2 + 1 = 16 + 1 = 17$$

$$-x^2 + 4x - 1 + 1 = 0$$

$$-x^2 + 4x = 0$$

$$x^2 - 4x = 0$$

$$x(x - 4) = 0$$

$$\downarrow \quad \downarrow$$

$$x = 0$$

$$x = 4$$

$$y = 1$$

$$y = 17$$

$$\{(0, 1), (4, 17)\}$$

Solve

Hint: Use Subs. method

$$\begin{cases} x - y = 3 \\ (x - 2)^2 + (y + 3)^2 = 4 \end{cases}$$

Isolate one variable

$$\rightarrow x = y + 3$$

$$x = -3 + 3 = 0$$

$$x = -1 + 3 = 2$$

$$(y + 3 - 2)^2 + (y + 3)^2 = 4$$

$$(y + 1)^2 + (y + 3)^2 = 4$$

$$(y + 1)(y + 1) + (y + 3)(y + 3) = 4$$

$$y^2 + y + y + 1 + y^2 + 3y + 3y + 9 - 4 = 0$$

$$2y^2 + 8y + 6 = 0$$

Divide by 2

$$y^2 + 4y + 3 = 0$$

$$(y + 3)(y + 1) = 0$$

$$\rightarrow y = -3$$

$$y = -1$$

$$x = 0$$

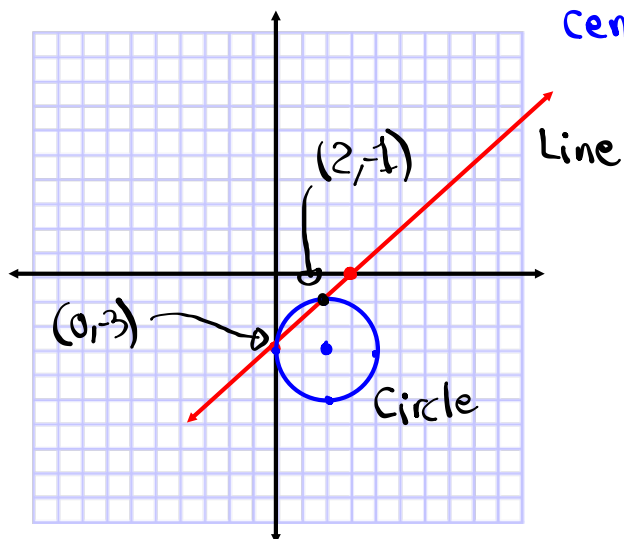
$$x = 2$$

$$\{(0, -3), (2, -1)\}$$

$$\begin{cases} x - y = 3 \longrightarrow \text{Line} \\ (x-2)^2 + (y+3)^2 = 4 \longrightarrow \text{Circle} \end{cases}$$

x	y
0	-3
3	0

center $(2, -3)$, $r=2$



Solve

$$\begin{cases} x + 2y = 0 \longrightarrow \text{Line} \\ (x-1)^2 + (y-1)^2 = 5 \longrightarrow \text{Circle} \end{cases}$$

$$x = -2y$$

$$(-2y-1)^2 + (y-1)^2 = 5$$

$$(-2y-1)(-2y-1) + (y-1)(y-1) = 5$$

$$4y^2 + 2y + 2y + 1 + y^2 - y - y + 1 - 5 = 0$$

$$5y^2 + 2y - 3 = 0$$

$$(5y - 3)(y + 1) = 0$$

$$5y - 3 = 0$$

$$y = \frac{3}{5}$$

$$y + 1 = 0$$

$$y = -1$$

$$x = -2y$$

$$x = -2\left(\frac{3}{5}\right) \quad x = -2(-1)$$

$$x = -\frac{6}{5} \quad x = 2$$

$$\left\{ \left(-\frac{6}{5}, \frac{3}{5}\right), (2, -1) \right\}$$

Given

$$(x - 2)^2 + (y + 4)^2 = 25$$

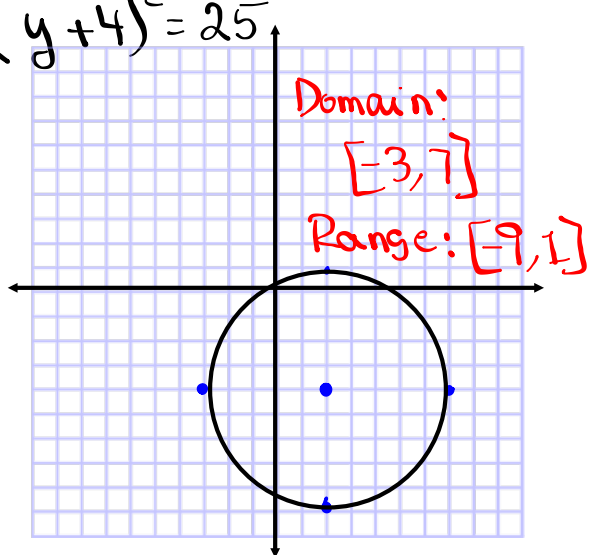
Center $(2, -4)$

Radius 5

Graph

Discuss

Domain & Range



Given: $\frac{(x+4)^2}{9} + \frac{(y-3)^2}{4} = 1$

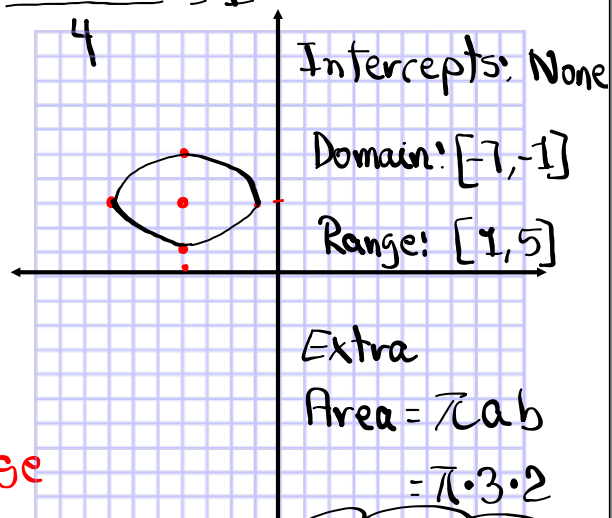
Center $(-4, 3)$

$$a^2 = 9 \quad a = 3$$

$$b^2 = 4 \quad b = 2$$

Draw

Discuss Domain & Range



Extra

$$\text{Area} = \pi ab$$

$$= \pi \cdot 3 \cdot 2$$

$$\text{Area} = 6\pi \text{ units}^2$$

Hyperbola

1) opens sideways

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

2) opens up & down

$$\frac{(y-k)^2}{b^2} - \frac{(x-h)^2}{a^2} = 1$$

Always

Always

Center (h, k)

Graph

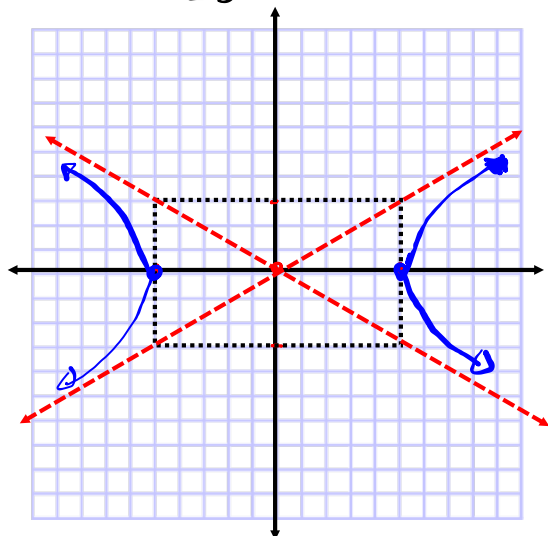
$$\frac{x^2}{25} - \frac{y^2}{9} = 1$$

opens sideways

Center $(0, 0)$

$$a^2 = 25 \quad a = 5$$

$$b^2 = 9 \quad b = 3$$



Graph

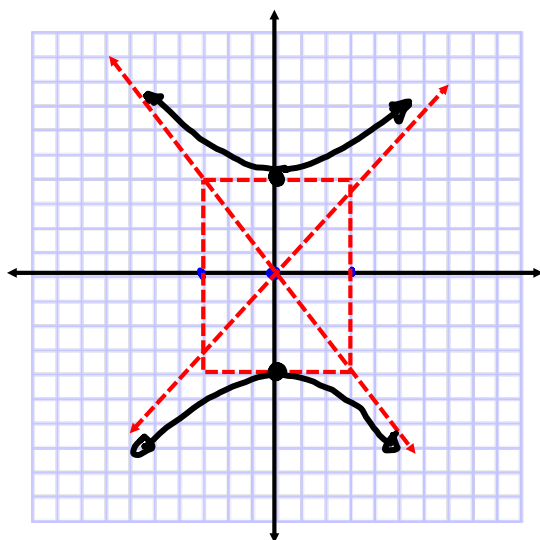
$$\frac{y^2}{16} - \frac{x^2}{9} = 1$$

Opens up & down

Center (0,0)

$$a^2 = 9 \quad a = 3$$

$$b^2 = 16 \quad b = 4$$



Graph

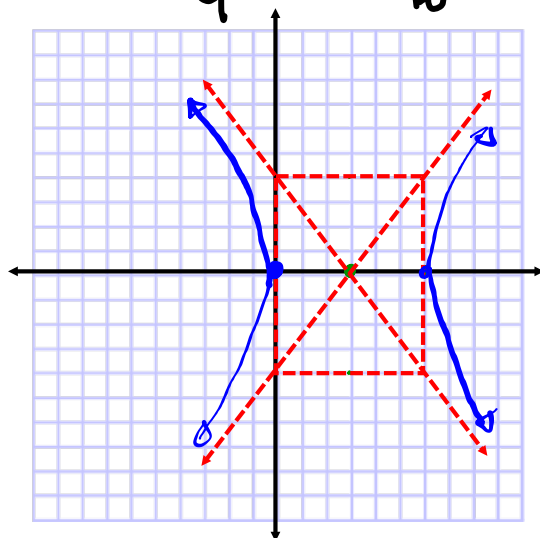
$$\frac{(x-3)^2}{9} - \frac{y^2}{16} = 1$$

Opens sideways

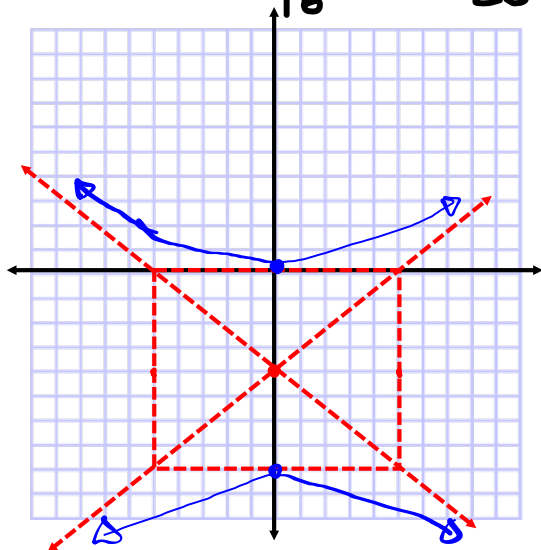
Center (3,0)

$$a^2 = 9 \quad a = 3$$

$$b^2 = 16 \quad b = 4$$



Graph $\frac{(y+4)^2}{16} - \frac{x^2}{25} = 1$ opens up & down



Center $(0, -4)$

$$a^2 = 25 \quad a = 5$$

$$b^2 = 16 \quad b = 4$$

x-Int: $(0, 0)$

Y-Ints: $(0, 0), (0, -8)$

Domain: $(-\infty, \infty)$

Range: $(-\infty, -8] \cup [0, \infty)$

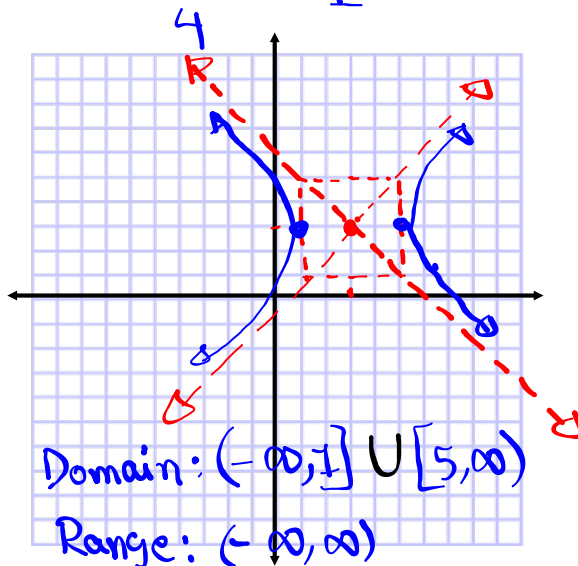
Graph $\frac{(x-3)^2}{4} - \frac{(y-3)^2}{4} = 1$

opens sideways

Center $(3, 3)$

$$a^2 = 4 \quad a = 2$$

$$b^2 = 4 \quad b = 2$$



Domain: $(-\infty, 1] \cup [5, \infty)$

Range: $(-\infty, \infty)$

Graph $4(y-4)^2 - 25(x+3)^2 = 100$

Hint: make RHS=1

Divide by 100, and Simplify

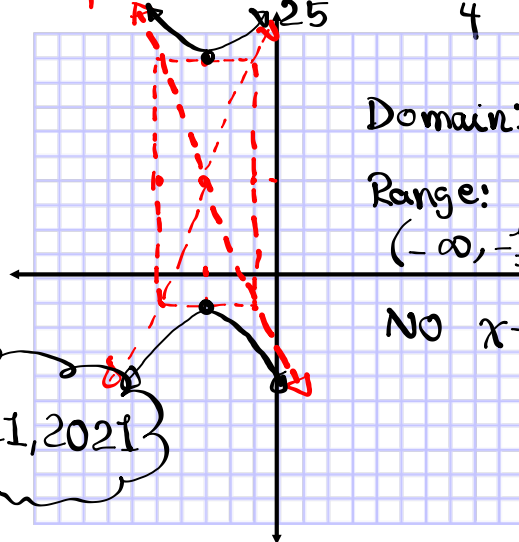
$$\frac{(y-4)^2}{25} - \frac{(x+3)^2}{4} = 1$$

Center $(-3, 4)$

$$a^2 = 4 \quad a = 2$$

$$b^2 = 25 \quad b = 5$$

opens up & down



Domain: $(-\infty, \infty)$

Range: $(-\infty, -1] \cup [9, \infty)$

NO x-Int.

NO School

&

Nov. 11, 2021

NO Class